Action Ward Identity and the Stückelberg-Petermann renormalization group*

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Summary. A fresh look at the renormalization group (in the sense of Stückelberg-Petermann) from the point of view of algebraic quantum field theory is given, and it is shown that a consistent definition of local algebras of observables and of interacting fields in renormalized perturbative quantum field theory can be given in terms of retarded products. The dependence on the Lagrangian enters this construction only through the classical action. This amounts to the commutativity of retarded products with derivatives, a property named Action Ward Identity by Stora.

1 Introduction

Modern perturbative quantum field theory is mainly based on path integrals. Basic objects are the correlation functions

$$G(x_1,\dots,x_n)=(\Omega,T\varphi(x_1)\cdots\varphi(x_n)\Omega) \tag{1}$$

which are calculated as moments of the Feynman path integral, i.e.

$$G(x_1,\dots,x_n)=\frac{1}{Z}\int\mathcal{D}\varphi\,\varphi(x_1)\cdots\varphi(x_n)e^{iS(\varphi)/\hbar}\ . \tag{2}$$

Relations to Wightman fields or even to local algebras of observables are indirect and are not elaborated in typical cases. This fact leads to some severe problems. On the one hand side, concepts from the algebraic approach to quantum field theory cannot be easily applied to perturbatively constructed models. This lead to the prejudice that these concepts are irrelevant for physically interesting models as long as these can be constructed only at the level of formal perturbation theory. On the other side, the path integral approach is intrinsically nonlocal; therefore difficulties arise for the treatment of infrared

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problems and of finite temperatures, and theories on curved back grounds or with external fields cannot properly be formulated. In algebraic quantum field theory, concepts for dealing with these problems have been developed [13, 2, 4, 14, 5, 17].

We therefore started a program for a perturbative construction of the net of local algebras of observables [4, 7]. There already exists a local formulation of renormalization due to Epstein and Glaser [11] and based on older ideas of Stückelberg and Bogoliubov [1]. The advantages of this method compared to other schemes of renormalization are that it can be formulated entirely in position space, that it is mathematically well elaborated and that it gives a direct construction of operators. It is, however, not equally well developed from the computational point of view, the rôle of the renormalization group is not yet fully established and, even worse, the application to non-Abelian gauge theories is not evident.

In particular the last problem was partially resolved by the Zürich school [19, 20], who also developed many new tools for doing concrete calculations in the Epstein-Glaser framework.

The starting point for our approach is Bogoliubov's definition of interacting fields

$$\varphi_{\int \mathcal{L}g}(x) = (Te^{i\int \mathcal{L}g})^{-1} \frac{\delta}{\delta h(x)} Te^{i(\int \mathcal{L}g + \int \varphi h)}|_{h=0} \tag{3}$$

In this contribution, we will first formulate axioms for interacting fields, will then construct solutions, thereafter discuss the renormalization group and will finally describe the arising local nets and local fields.

2 Basic properties required for interacting fields

We consider polynomial functionals of a classical scalar ($\mathcal{C}^\infty$) field φ on d-dimensional Minkowski space,

$$F(\varphi) = \sum_n \langle f_n, \varphi^{\otimes n} \rangle \ . \tag{4}$$

Here the smearing functions f_n are assumed to be distributions with compact support in n variables with wave front set

$$WF(f_n) \subset \{(x,k)\ ,\ \sum_i k_i = 0\} \tag{5}$$

e. g. $f_2(x_1,x_2) = g(x_1)\delta(x_1 - x_2)$ with $g \in \mathcal{D}$, so $\langle f_2, \varphi^{\otimes 2}\rangle = \int dx\, g(x)\varphi(x)^2$.

A functional F is called local, if $\mathrm{supp}(f_n) \subset D_n$, with the total diagonal

$$D_n = \{(x_1,\ldots,x_n)\ ,\ x_1 = \ldots = x_n \in \mathbf{R}^d\} \ . \tag{6}$$

Let us first describe the classical free theory: There the field equation $(\Box + m^2)\varphi = 0$ generates an ideal in the algebra of functionals F (with respect to pointwise multiplication). The Poisson bracket of the classical model is

$$\{F,G\}(\varphi) = \int dxdy \frac{\delta F}{\delta\varphi(x)} \Delta(x-y) \frac{\delta G}{\delta\varphi(y)} \tag{7}$$

where Δ is the commutator function of the free scalar field with mass m.

For quantization we follow the recipes of deformation quantization and define an associative product $*$ by

$$F * G(\varphi) = e^{\hbar \int dxdy \Delta_+(x-y) \frac{\delta}{\delta\varphi(x)} \frac{\delta}{\delta\varphi'(y)}} F(\varphi) G(\varphi')|_{\varphi'=\varphi} \tag{8}$$

where Δ_+ is the 2-point function of the free scalar field.

The Poisson bracket as well as the $*$-product vanish on the ideal generated by the field equation and can therefore be defined also on the quotient algebra. In the classical case one obtains the Poisson algebra of the classical field theory, in the quantum case one obtains the algebra of Wick polynomials on Fock space, and the formula for the $*$-product translates into Wick's Theorem. For the treatment of interactions, it is however preferable not to go to the quotient but to formulate everything on the original space (off shell formalism). This introduces some redundancy which will be very useful for the solution of cohomological problems, as for instance in the determination of counter terms in the Lagrangian which compensate changes in the renormalization prescription.

We now turn to the characterization of retarded interacting field functionals. Let F, S_n, $n \in \mathbf{N}$ be local functionals of φ. Let $S(\lambda) = \sum_{n=1}^{\infty} \lambda^n S_n$ be a formal power series with vanishing term of zeroth order. We associate to the pair $F, S(\lambda)$ a formal power series of functionals F_S which we interpret as the functional F of the interacting retarded field under the influence of the interaction S where λ is the expansion parameter of the formal power series.

We require the following properties:

Initial condition:

$$F_{S=0} = F \tag{9}$$

Causality:

$$F_{S_1+S_2} = F_{S_1} \tag{10}$$

if S_2 takes place later than F.

Glaser-Lehmann-Zimmermann: [12]

$$\frac{i}{\hbar}[F_S, G_S]_\star = \frac{d}{d\mu}(F_{S+\mu G} - G_{S+\mu F})|_{\mu=0} \ . \tag{11}$$

In addition, we require

Unitarity:

$$(F_S)^* = F^*_{S^*} \tag{12}$$

Covariance: Let β denote the natural action of the Poincaré group on the space of functionals F. Then

$$\beta_L(F_S) = \beta_L(F)_{\beta_L(S)} \tag{13}$$

for all Poincaré transformations L.

Field independence: The association of local functionals to their interacting counterpart should not explicitly depend on φ,

$$F_S(\varphi+\psi) = F(\varphi+\psi)_{S(\varphi+\psi)} \tag{14}$$

for test functions ψ.

Scaling: There is a natural scaling transformation, $x \mapsto \rho x$, on the space of functionals which scales also the mass in the $*$-product from m to $\rho^{-1}m$. But the limit $\rho \to \infty$ is singular because of scaling anomalies. This holds already for the free theory (in even dimensions).
Instead: Introduce an auxiliary mass parameter $\mu > 0$ and transform the $*$-product to an equivalent one which is smooth in the mass m. Require a smooth m dependence in the prescription $(F,S) \to F_S^{(m,\mu)}$ (in particular at $m = 0$ (one-sided)) such that $F_S^{(\rho^{-1}m,\mu)}$, scaled by ρ, is in every order of perturbation theory a polynomial of $\log \rho$.

3 Construction of solutions

We make the ansatz that the interacting field is a formal power series in the interaction,

$$F_{\lambda S} = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} R_{n,1}(S^{\otimes n} \otimes F) \ , \tag{15}$$

where $R_{n,1}$ is an $(n+1)$-linear functional, symmetric in the first n entries (called 'retarded product').

The inductive construction of retarded products can be done in an analogous way as the construction of time ordered products in the Epstein-Glaser method and was elaborated already (in a slightly different form) by Steinmann [21]. In a first step we represent the local functionals as fields smeared with a test function,

$$F(\varphi) = \int d^d x \, A(x) f(x) \tag{16}$$

where A is an element of the space $\mathcal{P}$ of all polynomials of φ and its derivatives,

$$A(x) = A(\varphi(x), \partial\varphi(x), \ldots) \ , \tag{17}$$

and f is a test function. Using this representation, the retarded products can be represented as distributions $R_{n,1}(A_1(x_1), \ldots, A_{n+1}(x_{n+1}))$ with values in the space of functionals of φ. With that the inductive construction can be done by first constructing the retarded products outside of the total diagonal D_{n+1} (6) in terms of retarded products with less arguments and then by extending them to the full space (in the sense of distributions). Perturbative renormalization is precisely this extension problem.

A difficulty with this procedure is that the representation of local functionals by fields is non-unique, e.g. one finds by partial integration

$$\int dx\, \partial A(x) f(x) = -\int dx\, A(x)\partial f(x)\ . \tag{18}$$

Therefore it was suggested by Stora [23] that one should impose an additional requirement, termed the **Action Ward Identity** (AWI), which guarantees that the retarded products depend only on the functionals, but not on the way they are represented as smeared fields. This amounts to the requirement, that the retarded products commute with derivatives,

$$\partial R_{n,1}(\ldots, A(x), \ldots) = R_{n,1}(\ldots, \partial A(x), \ldots)\ . \tag{19}$$

Such a relation cannot hold if the arguments of the retarded products are on-shell fields (i.e. satisfy the free field equations). In the off-shell formalism adopted here, however, one can actually show that there are no anomalies for the AWI. Namely one may introduce **balanced** fields, as was done recently for the purpose of non-equilibrium quantum field theory by Buchholz, Ojima and Roos [6]: the balanced fields form that subspace $\mathcal{P}_{\mathrm{bal}}$ of all local fields

$$A(x) = P(\partial_1, \ldots, \partial_n)\varphi(x_1)\cdots\varphi(x_n)|_{x_1=\ldots=x_n=x} \in \mathcal{P} \tag{20}$$

(where P is a polynomial) which arises when P is restricted to depend only on the differences of variables $(\partial_i - \partial_j)$.

The argument relies on

Lemma 1. *Every local functional F is of the form*

$$F = \int d^d x\, f(x) \tag{21}$$

with a unique test function f with values in the space of balanced fields.

Proof. Existence: Every symmetric polynomial $P(p_1, \ldots, p_n)$ in n variables $p_1, \ldots, p_n$, $p_i \in \mathbf{R}^d$ may be written in the form

$$P(p_1, \ldots, p_n) = \sum_j a_j(p) b_j(p_{\mathrm{rel}}) \tag{22}$$

with polynomials a_j in the center of mass momentum $p = \sum p_i$ and symmetric polynomials b_j in the relative momentum $p_{\mathrm{rel}} = (p_i - p_j, i < j)$. Every local functional of n-th order in φ is of the form

$$F(\varphi) = \int d^d x \sum_j \lambda_j(x) \tag{23}$$

$$\times \int d^{d(n-1)}x_{\mathrm{rel}}\, \delta(x_{\mathrm{rel}}) a_j(i\partial)\, b_j(i\partial_{rel})\varphi^{\otimes n}(x, x_{\mathrm{rel}}) \tag{24}$$

with test functions λ_j. By partial integration we find

$$F(\varphi) = \int dx \sum_j \mu_j(x) B_j(x) \tag{25}$$

with $\mu_j = a_j(-i\partial)\lambda_j$ and $B_j = b_j(i\partial_{\mathrm{rel}})\varphi^{\otimes n}|_{x_{\mathrm{rel}}=0} \in \mathcal{P}_{\mathrm{bal}}$.
Uniqueness: To show uniqueness let

$$F(\varphi) = \int dx \sum_j \mu_j(x) B_j(x) \tag{26}$$

with the balanced fields $B_j = b_j(i\partial_{\mathrm{rel}})\varphi^{\otimes n}|_{x_{\mathrm{rel}}=0}$, where b_j are *symmetrical* polynomials (with respect to permutations of $\partial_1, ..., \partial_n$). Then $F(\varphi) = 0$ implies that

$$\sum_j \mu_j(x) b_j(-i\partial_{\mathrm{rel}})\delta(x_{\mathrm{rel}}) = 0 \ . \tag{27}$$

Since the center of mass coordinate x and the relative coordinate x_{rel} are independent, this implies

$$\sum_j \mu_j \otimes b_j = 0 \ , \tag{28}$$

hence $\sum_j \mu_j B_j = 0$. $\square$

By using this Lemma the AWI can be fulfilled by the following procedure. One first extends $R(A_1(x_1), ..., A_{n+1}(x_{n+1}))$ only for $A_1, ..., A_{n+1} \in \mathcal{P}_{\mathrm{bal}}$. Since, by induction, the AWI holds outside of D_{n+1}, one may then define the extension for general fields $A_1, ..., A_{n+1}$ by using the AWI and linearity in the fields.

4 Renormalization group

The extension of $R_{n,1}$ from $\mathcal{D}(\mathbf{R}^{d(n+1)} \setminus D_{n+1})$ to $\mathcal{D}(\mathbf{R}^{d(n+1)})$ is generically non-unique. Hence, there is an ambiguity in the construction of interacting fields, which is well understood: it can be described in terms of the Stückelberg-Petermann renormalization group $\mathcal{R}$ [24].

The elements of $\mathcal{R}$ are analytic invertible maps $Z : S \mapsto Z(S)$ which map the space of formal power series of local functionals, which start with the first term, into itself such that

$$Z(0) = 0 \ , \tag{29}$$

$$Z'(0) = \mathrm{id} \qquad (\text{where } Z'(S)F := \frac{d}{d\tau} Z(S + \tau F)|_{\tau=0}) \ , \tag{30}$$

$$Z(S^\star) = Z(S)^\star \tag{31}$$

and

$$Z(S) = \int d^d x \, z(\mathcal{L}(x), \partial\mathcal{L}(x), \ldots) \tag{32}$$

if $S = \int dx\, \mathcal{L}(x)$. Here the Lagrangians are of the form $\mathcal{L}(x) = \sum_j A_j(x)g_j(x)$ with $A_j \in \mathcal{P}_{\mathrm{bal}}$ and $g_j \in \mathcal{D}(\mathbf{R}^d)$. Derivatives are defined by $\partial\mathcal{L} := \sum_j A_j(x)\partial g_j$. z is of the form

$$z(\mathcal{L}(x), \partial\mathcal{L}(x), \dots) = \sum_{n,a} \frac{1}{n!}\, d_{n,a}(A_{j_1} \otimes ... \otimes A_{j_n})(x) \prod_{i=1}^{n} \partial^{a_i} g_{j_i}(x) \qquad (33)$$

with linear maps $d_{n,a} : \mathcal{P}_{\mathrm{bal}}^{\otimes n} \to \mathcal{P}_{\mathrm{bal}}$ which are Lorentz invariant, maintain homogeneous scaling of the fields and do not explicitly depend on φ.

The Main Theorem of Renormalization [22, 18, 9] amounts to the following relation between interacting field functionals for two renormalization prescriptions $(F, S) \to F_S$ and $(F, S) \to \hat{F}_S$ which both satisfy the mentioned axioms:

Theorem 1. *There is a unique element Z of the renormalization group $\mathcal{R}$ such that*

$$\hat{F}_S = (Z'(S)F)_{Z(S)} \ . \qquad (34)$$

Conversely, given a renormalization prescription F_S satisfying the axioms and an arbitrary $Z \in \mathcal{R}$, equation (34) gives a new renormalization prescription $\hat{F}_S$ which fulfills also the axioms.

5 Local nets and local fields

We first define the algebra $\mathcal{A}_{\mathcal{L}}(\mathcal{O})$ of observables within the region $\mathcal{O}$ for a (fixed) interaction $\mathcal{L}$ with compact support. This algebra is generated by elements F_S, $S = \int dx\, \mathcal{L}(x)$ with local functionals F fulfilling supp $\frac{\delta F}{\delta\varphi} \subset \mathcal{O}$. In [4] it has been found that the algebraic structure of $\mathcal{A}_{\mathcal{L}}(\mathcal{O})$ is independent of the values of $\mathcal{L}$ outside of $\mathcal{O}$:

Theorem 2. *If the interactions $\mathcal{L}_1$ and $\mathcal{L}_2$ coincide within $\mathcal{O}$, there exist isomorphisms $\alpha : \mathcal{A}_{\mathcal{L}_1}(\mathcal{O}) \to \mathcal{A}_{\mathcal{L}_2}(\mathcal{O})$ such that*

$$\alpha(F_{S_1}) = F_{S_2} \qquad (35)$$

for all F localized in $\mathcal{O}$ (where $S_j = \int dx\, \mathcal{L}_j(x)$).

We now want to construct the algebras for a not necessarily compactly supported Lagrangian $\mathcal{L}$ (i.e. we perform the so called algebraic adiabatic limit). For this purpose we consider the bundle of algebras $\mathcal{A}_{\mathcal{L}_1}(\mathcal{O})$ over the space of compactly supported Lagrangians $\mathcal{L}_1$ which coincide within $\mathcal{O}$ with the given Lagrangian $\mathcal{L}$.

A section $B = (B_{\mathcal{L}_1})$ of this bundle is called covariantly constant if for all automorphisms α satisfying (35) the following relation holds:

$$\alpha(B_{\mathcal{L}_1}) = B_{\mathcal{L}_2} \ . \qquad (36)$$

So in particular the interacting field functionals F_S are (by definition) covariantly constant sections. The covariantly constant sections now generate the local algebras associated to the interaction $\mathcal{L}$.

To define the net of local algebras one has to specify, for $\mathcal{O}_1 \subset \mathcal{O}_2$, the injections

$$i_{\mathcal{O}_2\mathcal{O}_1} : \mathcal{A}_{\mathcal{L}}(\mathcal{O}_1) \to \mathcal{A}_{\mathcal{L}}(\mathcal{O}_2) \ . \tag{37}$$

Let $B \in \mathcal{A}_{\mathcal{L}}(\mathcal{O}_1)$. Then $i_{\mathcal{O}_2\mathcal{O}_1}(B)$ is the section which is obtained from the section B by restriction to Lagrangians which coincide with $\mathcal{L}$ on the larger region. Clearly these injections satisfy the compatibility relation required for nets:

$$i_{\mathcal{O}_3\mathcal{O}_2} \circ i_{\mathcal{O}_2\mathcal{O}_1} = i_{\mathcal{O}_3\mathcal{O}_1} \tag{38}$$

for $\mathcal{O}_1 \subset \mathcal{O}_2 \subset \mathcal{O}_3$. Moreover, the net satisfies local commutativity and, if $\mathcal{L}$ is Lorentz invariant, it is covariant under the Poincaré group [7].

The next step is the construction of local fields associated to the local net. Following [5], a local field associated to the net is defined as a family of distributions $(A_{\mathcal{O}})$ with values in $\mathcal{A}_{\mathcal{L}}(\mathcal{O})$ such that

$$i_{\mathcal{O}_2\mathcal{O}_1}(A_{\mathcal{O}_1}(h)) = A_{\mathcal{O}_2}(h) \tag{39}$$

if the test function h has support contained in $\mathcal{O}_1$. In particular all classical fields $A \in \mathcal{P}$ induce local fields $A^{\mathcal{L}} \equiv (A^{\mathcal{L}}_{\mathcal{O}})_{\mathcal{O}}$ by the sections

$$A^{\mathcal{L}}_{\mathcal{O}}(h) \ : \ \mathcal{L}_1 \mapsto (A_{\mathcal{O}}(h))_{\mathcal{L}_1} = (\int dx\, A(x)h(x))_{S_1} \tag{40}$$

where $\mathcal{L}_1$ coincides with $\mathcal{L}$ within $\mathcal{O}$, $h \in \mathcal{D}(\mathcal{O})$ and $S_1 = \int dx\, \mathcal{L}_1$. It is an open question whether there are other local fields. The answer would amount to the determination of the Borchers class for perturbative quantum field theories.

We may now restrict the renormalization group flow to constant Lagrangians $\mathcal{L} \in \mathcal{P}_{\mathrm{bal}}$.

Theorem 3. *Let $\mathcal{L} \mapsto \mathcal{A}_{\mathcal{L}}$ and $\mathcal{L} \mapsto \hat{\mathcal{A}}_{\mathcal{L}}$ ($\mathcal{L} \in \mathcal{P}_{\mathrm{bal}}$) be associations of local nets which are defined by the two renormalization prescriptions F_S and $\hat{F}_S$, respectively.*

1. *Then there exists a unique map ('renormalization of the interaction')*

$$z_0 : \lambda\mathcal{P}_{\mathrm{bal}}[[\lambda]] \longrightarrow \lambda\mathcal{P}_{\mathrm{bal}}[[\lambda]] \ : \ z_0(\lambda\mathcal{L}) = \lambda\mathcal{L} + O(\lambda^2). \tag{41}$$

such that the nets $\hat{\mathcal{A}}_{\mathcal{L}}$ and $\mathcal{A}_{z_0(\mathcal{L})}$ are equivalent for all $\mathcal{L}$.

2. *Furthermore there exists a unique map ('field renormalization')*

$$z^{(1)} : \lambda\mathcal{P}_{\mathrm{bal}}[[\lambda]] \times \mathcal{P}[[\lambda]] \longrightarrow \mathcal{P}[[\lambda]] \ : \ (\lambda\mathcal{L}, A) \mapsto z^{(1)}(\lambda\mathcal{L})A = A + O(\lambda) \tag{42}$$

such that $z^{(1)}(\lambda\mathcal{L}) : \mathcal{P}[[\lambda]] \longrightarrow \mathcal{P}[[\lambda]]$ is a linear invertible map which commutes with partial derivatives and such that for the local fields (40) the identification of $\hat{\mathcal{A}}_{\mathcal{L}}$ with $\mathcal{A}_{z_0(\mathcal{L})}$ is given by

$$\hat{A}^{\mathcal{L}} = (z^{(1)}(\mathcal{L})A)^{z_0(\mathcal{L})} \ , \quad \forall \mathcal{L} \in \mathcal{P}_{\mathrm{bal}}, \ A \in \mathcal{P} \ . \tag{43}$$

We point out that z_0 and $z^{(1)}$ are independent of $\mathcal{O}$.

Proof. By applying the algebraic adiabatic limit to Theorem 2 the formula (34) goes over into (43) with

$$z_0(\mathcal{L}) = z(\mathcal{L},0) = \sum_n \frac{1}{n!}\, d_{n,0}(\mathcal{L}^{\otimes n}) \tag{44}$$

and

$$z^{(1)}(\mathcal{L})A = \sum_{n,a\in\mathbf{N}_0^d} \frac{1}{n!}\,(-1)^{|a|}\,\partial^a d_{n+1,(0,\ldots,0,a)}(\mathcal{L}^{\otimes n}\otimes A)\ . \tag{45}$$

For the equivalence of the nets we refer to Theorem 5.1 of [9]. □

Example: $\mathcal{L} = \varphi^4$ in $d = 4$ dimensions. Then

$$z_0\mathcal{L} = (1+a)\varphi^4 + b((\partial\varphi)^2 - \varphi\Box\varphi) + m^2 c\varphi^2 + m^4 e\ , \tag{46}$$

where $a, b, c, e \in \mathbf{C}[[\lambda]]$. In the case $m = 0$ the coupling constant renormalization a and the field renormalization b are explicitly computed to lowest non-trivial order in [9].

Remarks. (1) It may happen that for a certain interaction $\mathcal{L}_0$ the renormalization is trivial, $z_0(\mathcal{L}_0) = \mathcal{L}_0$, but the corresponding field renormalization $z^{(1)}(\mathcal{L}_0)$ is non-trivial. This corresponds to the Zimmermann relations.

(2) The scaling transformations on a given renormalization prescription induce a one parameter subgroup of the renormalization group, which may be called a Gell-Mann-Low Renormalization Group. Its generator is related to the β-function. Gell-Mann-Low subgroups belonging to different renormalization prescriptions are conjugate to each other. The generator starts with a term of second order which is universal [3].

(3) An analysis of the perturbative renormalization group in the algebraic adiabatic limit was already given by Hollands and Wald in the more general framework of QFT on curved space times [15]. But the formalism presented here is not yet fully adapted to general Lorentzian space-times.

(4) Hollands and Wald generalized the Action Ward Identity to curved space times [16] (and called it 'Leibniz rule'). In that framework it is a non-trivial condition already for time ordered (or retarded) products of *one* factor.

(5) Ward identities, in particular the Master Ward Identity [10, 8] remain to be analyzed.

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